

CE10 · Refuting Graffiti 284

Capy Build task · grok-4-5 · 2026-07-22, 18:14–18:22 UTC · full reasoning traces, tool inputs, condensed outputs

PROMPT

You are hunting a FINITE COUNTEREXAMPLE to an open Graffiti spectral conjecture among {284, 292, 295} (all listed OPEN with girth ≥ 5 in Roucairol–Cazenave arXiv:2409.18626).

Task

1. Look up exact statements for Graffiti 284, 292, 295 from Aouchiche–Hansen / WotW.
2. Pick the one that looks MOST vulnerable to finite counterexample (simplest invariants, sharp on small cages).
3. Attack with code: gravity/distance/adjacency spectra on girth ≥ 5 graphs (Petersen, Heawood, Tutte–Coxeter, random (3,g)-graphs, incidence graphs of projective planes, etc.).
4. Use correct definitions (range = max-min vs number of distinct — document which).

DGG style: simple, relentless, exact certificate.

Deliverable

Which conjecture, statement, counterexample or best margin.

Work in /home/scout/conjecture-hunt/ce10-graffiti-girth/.

01 · 18:14

The user wants me to hunt for a finite counterexample to an open Graffiti spectral conjecture among {284, 292, 295}. Let me start by looking up the exact statements and setting up the work directory.

I'll look up the exact statements for Graffiti 284/292/295 and set up a counterexample hunt in the work directory.

\$ bash

```
mkdir -p /home/scout/conjecture-hunt/ce10-graffiti-girth && ls /home/scout/conjecture-hunt/ 2>/dev/null; ls /home/2>/dev/null | head -50
```

```
ce10-graffiti-girth
copy-site
e2b-core
scout
ubuntu
user
Command succeeded. Exit code: 0. Duration: 53 ms.
```

search · Graffiti conjecture 284 292 295 Roucairol Cazenave spectral girth

Web search results for "Graffiti conjecture 284 292 295 Roucairol Cazenave spectral girth":
[1] Refutation of Spectral Graph Theory Conjectures with Search Algorithms
URL: <https://www.lamsade.dauphine.fr/~cazenave/papers/ConjectureRefutationECAI2025.pdf>
Highlights: • Graffiti 294 from Aouchiche and Hansen's survey is Graffiti 295 from "written on the wall", we go ...
K3 - free means there is no clique of size 3, "girth ≥ 5 " means the shortest cycle must be longer than 5, "tree" ...
... 175 lines
Web search results for "Graffiti conjecture 284 29 ...

search · Aouchiche Hansen Graffiti 284 292 295 spectrum diameter gravity

Web search results for "Aouchiche Hansen Graffiti 284 292 295 spectrum diameter gravity":
[1] A Survey of Automated Conjectures in Spectral Graph Theory (Published: 2009-03-01T00:00:00.000Z)
URL: <https://gerad.ca/fr/papers/G-2009-18>
Author: Mustapha Aouchiche, Pierre Hansen
Highlights: Abstract During the last three decades, the computer has been widely used in spectral graph theory. Many results about graph eigenvalues were first conjectured, and in some cases proved, using computer programs, such as GRAPH, Graffiti, Ingrid, newGRAPH and AutoG ...

02 · 18:15

I need the exact statements of Graffiti 284, 292, 295. Let me fetch the arxiv paper and Aouchiche-Hansen survey, and also look for Written on the Wall.

Fetching the papers for exact conjecture statements.

```
fetch · https://ar5iv.labs.arxiv.org/html/2409.18626
```

```

  Fetched https://ar5iv.labs.arxiv.org/html/2409.18626 - [2409.18626] Refutation of Spectral Graph Theory Conjectures with
  Search Algorithms
  Content:
  [2409.18626] Refutation of Spectral Graph Theory Conjectures with Search Algorithms
  11institutetext: aLAMSADE, Université Paris-Dauphine, Paris, Pl. du Maréchal de Lattre de Tassigny, 75016 Paris, France
  # Refutation of Spectral Graph Theory Conjectures with Search Algorithms
  Milo Roucairol and Trist ...
```

```
fetch · https://arxiv.org/pdf/2409.18626
```

```

  Fetched https://arxiv.org/pdf/2409.18626 - Refutation of Spectral Graph Theory Conjectures with Search Algorithms)
  Content:
  # Refutation of Spectral Graph Theory Conjectures with Search Algorithms)
  arXiv (Cornell University), 2024. Preprint. 0 citations.
  ## Authors
  - Milo Roucairol: h-index 3; 27 citations
  ... 41 lines
  Fetched https://arxiv.org/pdf/2409.18626 - Refutation of Spectral Graph Theory Conjectures with Search Algorithms)
  Content:
  # Refut ...
```

```
$ bash
```

```
curl -sL "https://arxiv.org/pdf/2409.18626.pdf" -o /tmp/roucairol.pdf && pdftotext -layout /tmp/roucairol.pdf
/tmp/roucairol.txt 2>/dev/null; wc -l /tmp/roucairol.txt; rg -n -i "284|292|295|gravity|range|girth" /tmp/roucairol.txt |
head -80
```

```

wc: /tmp/roucairol.txt: No such file or directory
rg: /tmp/roucairol.txt: No such file or directory (os error 2)
Command succeeded. Exit code: 0. Duration: 303 ms.
```

03 · 18:15

I need the exact statements of conjectures 284, 292, 295 from Aouchiche-Hansen survey or Written on the Wall. Let me search for the github with WotW and the survey.

Need the exact WotW/Aouchiche-Hansen statements next.

```
search · "Written on the Wall" Graffiti Fajtlowicz 284 292 295 github Roucairol
```

```

Web search results for ""Written on the Wall" Graffiti Fajtlowicz 284 292 295 github Roucairol":
[1] Refutation of Spectral Graph Theory Conjectures with Search Algorithms
URL: https://www.lamsade.dauphine.fr/~cazenave/papers/ConjectureRefutationECAI2025.pdf
Highlights: "Written on the wall" collects almost 1
...
We provide a copy of "Written on the Wall" on this project's github, which is otherwise hard to find:
...
... 148 lines
Web search results for ""Written on the Wall" Graffiti Fajtlowicz 284 292 295 github Roucairol":
[1] Refutation of S ...
```

```
search · Aouchiche Hansen "survey of automated conjectures" graffiti 284 292 site:arxiv.org
```

```

Web search results for "Aouchiche Hansen "survey of automated conjectures" graffiti 284 292 site:arxiv.org":
[1] arXiv:2409.18626v1 [cs.AI] 27 Sep 2024 (Published: 2024-09-27T00:00:00.000Z)
URL: https://arxiv.org/pdf/2409.18626
Author: Milo Roucairol, Tristan Cazenave
Highlights: GraphiX
...
conjectures to verify
... 78 lines
Web search results for "Aouchiche Hansen "survey of automated conjectures" graffiti 284 292 site:arxiv.org":
[1] arXiv:2409.18626v1 [cs.AI] 27 Sep 2024 (Published: 2024-09-27T00:00:00.000Z)
URL: https://arxiv.org/pdf/2409.18 ...
```

```
$ bash
```

```
curl -sL "https://ar5iv.labs.arxiv.org/html/2409.18626" -o /tmp/r.html; rg -n "284|292|295|Graffiti 29" /tmp/r.html | head -40; ls /tmp/; which pdftotext; curl -sL "https://arxiv.org/pdf/2409.18626" -o /tmp/r.pdf && file /tmp/r.pdf && pdftotext - layout /tmp/r.pdf /tmp/r.txt && rg -n "284|292|295" /tmp/r.txt | head -40
```

```
398:<span id="S4.T1.2.p2.5.5.5.5.2" class="ltx_td ltx_align_left ltx_border_r">284 0</span>
578:<span id="S4.T1.2.p2.8.8.8.8.2" class="ltx_td ltx_align_left ltx_border_r">292 0</span>
590:<span id="S4.T1.2.p2.9.9.9.9.2" class="ltx_td ltx_align_left ltx_border_r">295 0</span>
712:<p id="S5.p2.1" class="ltx_p">Considering nonterminal states for evaluation helped immensely with the harder Graffiti
137, 139, 189, 289, and 301. Graffiti 289's repeating pattern may be an explanation as to why the NRPA succeeded where most
other algorithms failed. The times, when displayed in the hundreds of seconds, are approximations, only GBFS and BEAM are
deterministic so the time is exact. In addition, NMCS, UCT, GRAVE, and RAVE were very unreliable at finding a counter-example
for Graffiti 29, about once every 3 runs, when LNMCS was able to find a co --
```

04 · 18:15

There's a GitHub repo with the implementations: <https://github.com/RoucairolMilo/refutationGBR> or <https://github.com/RoucairolMilo/refutationExperimentalMathematics>. Let me clone that and also get the...

Cloning the Roucairol repo and pulling the Aouchiche–Hansen survey for exact statements.

\$ bash

```
cd /tmp && git clone --depth 1 https://github.com/RoucairolMilo/refutationGBR.git 2>&1 | tail -5; git clone --depth 1
https://github.com/RoucairolMilo/refutationExperimentalMathematics.git 2>&1 | tail -5; ls /tmp/refutationGBR 2>/dev/null; ls
/tmp/refutationExperimentalMathematics 2>/dev/null
```

```
Cloning into 'refutationGBR'...
Cloning into 'refutationExperimentalMathematics'...
remote: Repository not found.
fatal: repository 'https://github.com/RoucairolMilo/refutationExperimentalMathematics.git/' not found
Cargo.toml
src
Command failed. Exit code: 2. Duration: 835 ms.
```

\$ bash

```
curl -sL "https://www.lamsade.dauphine.fr/~cazenave/papers/ConjectureRefutationECAI2025.pdf" -o /tmp/ecai.pdf; file
/tmp/ecai.pdf; python3 -c "
import sys
try:
    import pypdf
    print('pypdf ok')
except: pass
try:
    import PyPDF2
    print('PyPDF2 ok')
except: pass
try:
    import fitz
    print('fitz ok')
except: pass
" 2>&1; pip install pypdf -q && python3 -c "
from pypdf import PdfReader
r=PdfReader('/tmp/roucairol.pdf')
text='\n'.join(p.extract_text() or '' for p in r.pages)
open('/tmp/r.txt', 'w').write(text)
print('pages', len(r.pages), 'chars', len(text))
for kw in ['284', '292', '295', '290', 'gravity', 'range', 'girth']:
    print('---', kw, text.lower().count(kw.lower()))
"
```

```
/tmp/ecai.pdf: PDF document, version 1.7, 6 page(s)
error: externally-managed-environment
× This environment is externally managed
↳ To install Python packages system-wide, try apt install
python3-xyz, where xyz is the package you are trying to
install.
```

```
If you wish to install a non-Debian-packaged Python package,
create a virtual environment using python3 -m venv path/to/venv.
Then use path/to/venv/bin/python and path/to/venv/bin/pip. Make
```

... 7 lines

note: If you believe this is a mistake, please contact your Python installation or OS distribution provider. You can override this, at the risk of breaking your Python installation or OS, by passing `--break-system-packages`.

hint: See PEP 668 for the detailed specification.
Command failed. Exit code: 1. Duration: 1902 ms.

05·18:15

Let me explore the cloned repo and install pypdf in a venv.

\$ bash

```
find /tmp/refutationGBR -type f | head -100; ls -la /tmp/refutationGBR/src/
```

```
/tmp/refutationGBR/.git/info/exclude
/tmp/refutationGBR/.git/description
/tmp/refutationGBR/.git/hooks/commit-msg.sample
/tmp/refutationGBR/.git/hooks/post-update.sample
/tmp/refutationGBR/.git/hooks/fsmonitor-watchman.sample
/tmp/refutationGBR/.git/hooks/pre-applypatch.sample
/tmp/refutationGBR/.git/hooks/pre-commit.sample
/tmp/refutationGBR/.git/hooks/pre-merge-commit.sample
/tmp/refutationGBR/.git/hooks/pre-push.sample
/tmp/refutationGBR/.git/hooks/pre-rebase.sample
... 43 lines
drwxr-xr-x 3 root root 60 Jul 22 18:15 models
drwxr-xr-x 2 root root 128 Jul 22 18:15 tools
Command succeeded. Exit code: 0. Duration: 11 ms.
```

\$ bash

```
python3 -m venv /tmp/venv && /tmp/venv/bin/pip install pypdf numpy networkx scipy -q && /tmp/venv/bin/python -c "
from pypdf import PdfReader
for path in ['/tmp/roucairol.pdf', '/tmp/ecai.pdf']:
    r=PdfReader(path)
    text='\n'.join(p.extract_text() or '' for p in r.pages)
    open(path.replace('.pdf', '.txt'), 'w').write(text)
    print(path, 'pages', len(r.pages), 'chars', len(text))
" && rg -n "284|292|295|290|Graffiti" /tmp/roucairol.txt /tmp/ecai.txt | head -80
```

```
/tmp/roucairol.pdf pages 14 chars 27644
/tmp/ecai.pdf pages 9 chars 26231
/tmp/roucairol.txt:14:search algorithms to a selection of conjectures from Graffiti. Out of 13
/tmp/roucairol.txt:15:already refuted conjectures from Graffiti, our algorithms are able to re-
/tmp/roucairol.txt:16:fute 12 in seconds. We also refute conjecture 197 from Graffiti which was
/tmp/roucairol.txt:31:Auto-GraphiX [9] and Graffiti [5], there are plenty of such conjectures.
/tmp/roucairol.txt:34:[16]. Wagner was able to refute three conjectures from Graffiti. Wagner's method
/tmp/roucairol.txt:71:address all the remaining conjectures from Graffiti that do not involve solving a
/tmp/roucairol.txt:72:NP-hard problem. Graffiti is a well-known software that was used to generate a
/tmp/roucairol.txt:79:We decided to focus on this subset of Graffiti conjectures to av ...
```

06·18:15

Let me look at the Rust source for the conjecture definitions - specifically invariants and GenerateGraph.

read · /tmp/refutationGBR/src/models/conjectures/invariants.rs

```
1 use nalgebra::{DMatrix, Dynamic};
2 use nalgebra::linalg::SymmetricEigen;
3
4 pub fn std_dev(data : Vec<f64>) → f64 {
5     let mut m : f64 = 0.0;
6     for i in 0..data.len(){
7         m += data[i];
8     }
... 293 lines
1 use nalgebra::{DMatrix, Dynamic};
2 use nalgebra::linalg::SymmetricEigen;
3
4 pub fn std_dev(data : Vec<f64>) → f64 {
5     let mut m : f64 = 0.0;
6     for i in 0..data.len(){
7         m += data[i];
8     }
9     m = m/(data.len() as f64);
10
11     let mut s : f64 = ...
```

```
read · /tmp/refutationGBR/src/models/conjectures/mod.rs
```

```
1 pub(crate) mod GenerateGraph;
2 pub(crate) mod invariants;
3
4
```

```
$ bash
```

```
rg -n "284|292|295|290|conjecture|gravity|range|girth" /tmp/refutationGBR/src -i | head -100; ls /tmp/refutationGBR/; cat /tmp/refutationGBR/Cargo.toml; head -100 /tmp/refutationGBR/src/main.rs
```

```
/tmp/refutationGBR/src/main.rs:8:use crate::models::conjectures::GenerateGraph::{State as StateGraph};
/tmp/refutationGBR/src/models/mod.rs:1:pub(crate) mod conjectures;
/tmp/refutationGBR/src/methods/lazyNMCSv3.rs:4:use crate::models::conjectures::GenerateGraph::{State};
/tmp/refutationGBR/src/methods/UCT.rs:3:use crate::models::conjectures::GenerateGraph::{State, Move};
/tmp/refutationGBR/src/methods/NRPA.rs:1:use crate::models::conjectures::GenerateGraph::{State, Move};
/tmp/refutationGBR/src/methods/NRPA.rs:4://use crate::models::conjectures::conjecture2p1::{State, Move};
/tmp/refutationGBR/src/models/conjectures/invariants.rs:117:pub fn gravity_matrix(adj_mat : &DMatrix<f64>) → DMatrix<f64>{
/tmp/refutationGBR/src/methods/NMCS.rs:2:use crate::models::conjectures::GenerateGraph::{State};
/tmp/refutationGBR/src/methods/NMCS.rs ...
```

```
07 · 18:15
```

I found the implementations for 284, 292, 295. Let me read those sections carefully and also get the Aouchiche-Hansen survey statements.

Reading the score functions for 284/292/295 and pulling the survey statements.

```
read · /tmp/refutationGBR/src/models/conjectures/GenerateGraph.rs
```

```
320 self.stored_score = lambda2 - mean_du + 100.0;
321 self.stored_score_pertinent = true;
322
323 return lambda2 - mean_du + 100.0
324 }
325
326 if CONJECTURE == 284 {
327
... 42 lines
320 self.stored_score = lambda2 - mean_du + 100.0;
321 self.stored_score_pertinent = true;
322
323 return lambda2 - mean_du + 100.0
324 }
325
326 if CONJECTURE == 284 {
327
328     if self.n_sommet < 3 {return 0.0}
329
330 ...
```

```
read · /tmp/refutationGBR/src/models/conjectures/GenerateGraph.rs
```

```
1600 self.stored_score_pertinent = true;
1601 //println!("{}", self.stored_score);
1602
1603 return inv_even - rank_dist + 100.0;
1604
1605 }
1606
1607 if CONJECTURE == 292 {
... 72 lines
1600 self.stored_score_pertinent = true;
1601 //println!("{}", self.stored_score);
1602
1603 return inv_even - rank_dist + 100.0;
1604
1605 }
1606
1607 if CONJECTURE == 292 {
1608
1609     // min_pos_eig ≤ n_sommet / meangravity
1610
16 ...
```

```
$ bash
```

```
# Get Aouchiche-Hansen survey
curl -sL "https://www.sciencedirect.com/science/article/pii/S0024379509003061" -o /tmp/ah.html 2>&1 | tail -3
# try GERAD version
curl -sL "https://www.gerad.ca/en/papers/G-2009-18" -o /tmp/gerad.html
rg -o 'href="[^\"]+\.pdf"' /tmp/gerad.html | head -20
# try direct pdf
for u in \
  "https://www.gerad.ca/papers/G-2009-18.pdf" \
  "https://www.gerad.ca/en/papers/G-2009-18.pdf" \
  "https://germain.hec.ca/pages/mustapha.aouchiche/G-2009-18.pdf" \
  "https://arxiv.org/pdf/math/0000000" \
  "https://www.cs.auckland.ac.nz/~mjd/graffiti/wow.pdf"; do
  code=$(curl -sL -o /tmp/try.pdf -w "%{http_code}" "$u")
  echo "$code $u $(file /tmp/try.pdf | head -c 80)"
done
# Written on the Wall from github of Roucairol
cd /tmp/refutationGBR && git ls-tree -r HEAD --name-only | head -50
# search github for WotW
curl -sL "https://api.github.com/repos/RoucairolMilo/refutationGBR/contents/" | head -c 2000
# search for written on the wall pdf
curl -sL "https://api.github.com/search/code?q=repo:RoucairolMilo/refutationGBR+284" | head -c 500
curl -sL "https://raw.githubusercontent.com/RoucairolMilo/refutationGBR/main/README.md" 2>/dev/null | head -50
ls -la /tmp/refutationGBR/
```

```
404 https://www.gerad.ca/papers/G-2009-18.pdf /tmp/try.pdf: ASCII text, with no line terminators
404 https://www.gerad.ca/en/papers/G-2009-18.pdf /tmp/try.pdf: ASCII text, with no line terminators
000 https://germain.hec.ca/pages/mustapha.aouchiche/G-2009-18.pdf /tmp/try.pdf: ASCII text, with no line terminators
404 https://arxiv.org/pdf/math/0000000 /tmp/try.pdf: HTML document, ASCII text
404 https://www.cs.auckland.ac.nz/~mjd/graffiti/wow.pdf /tmp/try.pdf: HTML document, ASCII text
Cargo.toml
src/main.rs
src/methods/BEAM.rs
src/methods/BFS.rs
src/methods/GRAVE.rs
... 56 lines
-rw-r--r--  1 root root  279 Jul 22 18:15 Cargo.toml
drwxr-xr-x  5 root root  128 Jul 22 18:15 src
Command succeeded. Exit code: 0. Duration: 2671 ms.
```

```
08:18:16
```

I have the implementations from Roucairol's code which maps to Aouchiche-Hansen. Let me also get the survey text for the exact statements. Looking at what we have:

Pulling Aouchiche-Hansen statements and implementing the spectral attack.

```
search · "mininum of the dual degree" OR "minimum dual degree" graffiti distance eigenvalue
```

```
Web search results for "\"mininum of the dual degree\" OR \"minimum dual degree\" graffiti distance eigenvalue\":
[1] Distance spectra of graphs: A survey
URL: https://www.math.ucdavis.edu/~saito/data/distmat/aouchiche-hansen_dist-spec-graphs-survey.pdf
Author: Mustapha Aouchiche
Highlights: the maximum and minimum degrees of
...
, respectively.
... 279 lines
Web search results for "\"mininum of the dual degree\" OR \"minimum dual degree\" graffiti distance eigenvalue\":
[1] Distance spectra of graphs: A survey
URL: https://www.math.ucdavis.edu/~saito/da ...
```

```
search · Aouchiche Hansen graffiti "girth at least 5" gravity positive eigenvalue 284 OR 292 OR 294
```

```
Web search results for "Aouchiche Hansen graffiti \"girth at least 5\" gravity positive eigenvalue 284 OR 292 OR 294\":
[1] Open problems on graph eigenvalues studied with AutoGraphiX - ScienceDirect (Published: 2013-05-01T00:33:17.000Z)
URL: https://www.sciencedirect.com/science/article/abs/pii/S2192440621000174
Highlights: Mustapha Aouchiche 1, Gilles Caporossi 1, Pierre Hansen 1
...
In this paper, we consider four open problems related to the eigenvalues of (the adjacency matrix of) a connected graph. The
problem is related to the maximum value ...
```

```
$ bash
```

```
# Try sci-hub / open access copies
for u in \
  "https://sci-hub.se/10.1016/j.laa.2009.06.015" \
  "https://sci-hub.st/10.1016/j.laa.2009.06.015" \
  "https://sci-hub.ru/10.1016/j.laa.2009.06.015" \
  "https://www.researchgate.net/publication/220655687_A_survey_of_automated_conjectures_in_spectral_graph_theory"; do
code=$(curl -sL -A "Mozilla/5.0" -o /tmp/ahtry.html -w "%{http_code}" --max-time 15 "$u")
echo "$code $u size=$(wc -c </tmp/ahtry.html)"
rg -o 'https?:/[^\"]+\.pdf' /tmp/ahtry.html 2>/dev/null | head -5
done
# also try mustapha aouchiche homepage
curl -sL "https://www.gerad.ca/en/papers/G-2009-18" -A "Mozilla/5.0" | rg -i "pdf|download|284|gravity" | head -30
curl -sL "https://www.gerad.ca/fr/papers/G-2009-18" -A "Mozilla/5.0" | rg -i "pdf|href" | head -40
```

Bash could not parse this command, so nothing was executed:
command: line 17: unexpected EOF while looking for matching ``'
Command failed. Duration: 4 ms.

09 · 18:16

Found Written on the Wall PDF: <https://independencenumber.wordpress.com/wp-content/uploads/2012/08/wow-july2004.pdf>

Found WotW PDF — extracting 284/292/295.

\$ bash

```
curl -sL "https://independencenumber.wordpress.com/wp-content/uploads/2012/08/wow-july2004.pdf" -o /home/scout/conjecture-hunt/ce10-graffiti-girth/wow-july2004.pdf && file /home/scout/conjecture-hunt/ce10-graffiti-girth/wow-july2004.pdf && /tmp/venv/bin/pip install pypdf -q && /tmp/venv/bin/python << 'PY'
from pypdf import PdfReader
r=PdfReader('/home/scout/conjecture-hunt/ce10-graffiti-girth/wow-july2004.pdf')
print('pages', len(r.pages))
# extract all text
texts=[]
for i,p in enumerate(r.pages):
    t=p.extract_text() or ''
    texts.append(t)
full='\n'.join(texts)
open('/home/scout/conjecture-hunt/ce10-graffiti-girth/wow.txt', 'w').write(full)
print('chars', len(full))
# find conjecture numbers near 284 etc
import re
for m in re.finditer(r'(?i)(conjecture\s*)?(2[89]\d|3\d\d)\b', full):
    start=max(0,m.start()-80); end=min(len(full), m.end()+200)
    snip=full[start:end].replace('\n', ' ')
    if any(x in snip.lower() for x in ['284','292','294','295','290','gravity','girth','dual']):
        print('---', m.group(), '---')
        print(snip)
        print()
```

/home/scout/conjecture-hunt/ce10-graffiti-girth/wow-july2004.pdf: PDF document, version 1.2, 10 page(s)
pages 216
chars 956088
Command succeeded. Exit code: 0. Duration: 1808 ms.

10 · 18:16

The regex might have failed to find matches. Let me search more carefully.

\$ bash

```
/tmp/venv/bin/python << 'PY'
full=open('/home/scout/conjecture-hunt/ce10-graffiti-girth/wow.txt').read()
print(repr(full[:500]))
# search for 284
for needle in ['284','292','294','295','290','Gravity','gravity','girth']:
    idx=full.find(needle)
    print(needle, 'first', idx)
# try bytes/encoding issues - look for lines with numbers
import re
# Find patterns like "284." at start of lines or after newlines
matches=list(re.finditer(r'(?:\n)\s*({2,3})[.]\s', full))
```

```

print('numbered items', len(matches))
# print some around 280-300
for m in matches:
    num=int(m.group(1))
    if 270<=num<=330:
        start=m.start(); end=min(len(full), start+400)
        print('===', num, '===')
        print(full[start:end].replace('\n',' | '))
        print()

```

PY

```

'/CF /D6/CX/D8/D8/CT/D2 /D3/D2 /D8/CW/CT /CF /CP/D0/D0\n/D2 /D1 /CQ /D6 /CS /D1 /D7 /D6 /CS /CS /DA /CS /CS\n/CC/CW/CX/D7
/D0/CX/D7/D8 /CR /D3/D2/D8/CP/CX/D2/D7 /D1/D3/D6 /CT /D8/CW/CP/D2 /CW/CP/D0/CU /D3/CU /BZ/D6 /CPÆ/D8/CX/B3/D7 /CR
/D3/D2/CY/CT /CR/D8/D9/D6 /CT/D7 /CU/D6 /D3/D1 /D8/CW/CT /D3/D6/CX/CV/CX/D2/CP/D0 /DA/CT/D6/B9\n/D7/CX/D3/D2 /D3/CU
A/CF/D6/CX/D8/D8/CT/D2 /D3/D2 /D8/CW/CT /CF/CP/D0 /D0A /CP/D2/CS /D7/D3/D1/CT /D2/CT/DB/CT/D6 /D3/D2/CT/D7/B8 /CQ/D9/D8
/CT/DA/CT/D2/D8/D9/CP/D0 /D0/DD '
284 first -1
292 first -1
294 first -1
295 first -1
290 first -1
Gravity first -1
gravity first -1
girth first -1
numbered items 0
Command succeeded. Exit code: 0. Duration: 28 ms.

```

11·18:16

The PDF extraction failed due to encoding - it's a weird encoded PDF. Let me try pdftotext or ocrmypdf or another approach.

PDF text is encoded oddly — trying alternative extraction.

\$ bash

```

sudo apt-get install -y poppler-utils 2>&1 | tail -5
pdftotext -layout /home/scout/conjecture-hunt/ce10-graffiti-girth/wow-july2004.pdf /tmp/wow2.txt 2>&1
file /tmp/wow2.txt; head -c 500 /tmp/wow2.txt; echo; wc -l /tmp/wow2.txt
# check if binary garbage
python3 -c "
t=open('/tmp/wow2.txt','rb').read(200)
print(t[:100])
"
# try pdfminer
/tmp/venv/bin/pip install pdfminer.six -q
/tmp/venv/bin/python -c "
from pdfminer.high_level import extract_text
t=extract_text('/home/scout/conjecture-hunt/ce10-graffiti-girth/wow-july2004.pdf', maxpages=5)
print(repr(t[:800]))
print('len',len(t))
"

```

Selecting previously unselected package poppler-utils.

```

(Reading database ...
(Reading database ... 5%
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(Reading database ... 15%
(Reading database ... 20%
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(Reading database ... 75%
(Reading database ... 80%
(Reading database ... 85%
(Reading database ... 90%
(Reading database ... 95%
(Reading database ... 100%

```

```
(Reading database ... 169171 files and directories currently installed.)
Preparing to unpack .../poppler-utils_24.02.0-1ubuntu9.9_amd64.deb ...
Unpacking poppler-utils (24.02.0-1ubuntu9.9) ...
Setting up poppler-utils (24.02.0-1ubuntu9.9) ...
/tmp ...
```

12 · 18:16

The PDF uses a custom encoding. Let me try OCR on rendered pages, or look at the Aouchiche-Hansen paper differently. Also Brewster et al. might have gravity matrix definitions with these conjectures.

...

Custom font encoding — fetching Aouchiche–Hansen and Brewster via other routes, and implementing the attack from Roucairol’s score functions.

\$ bash

```
# OCR approach on a few pages around likely conjecture numbers - first render PDF pages
sudo apt-get install -y tesseract-ocr 2>&1 | tail -3
# Convert pages - WotW has 216 pages. Find which pages have 284 etc via OCR sampling
pdftoppm -f 60 -l 70 -r 150 /home/scout/conjecture-hunt/ce10-graffiti-girth/wow-july2004.pdf /tmp/wowp
ls /tmp/wowp* | head
tesseract /tmp/wowp-1.png stdout 2>/dev/null | head -40
```

```
Setting up libtesseract5:amd64 (5.3.4-1build5) ...
Setting up tesseract-ocr (5.3.4-1build5) ...
Processing triggers for libc-bin (2.39-0ubuntu8.7) ...
/tmp/wowp-060.ppm
/tmp/wowp-061.ppm
/tmp/wowp-062.ppm
/tmp/wowp-063.ppm
/tmp/wowp-064.ppm
/tmp/wowp-065.ppm
/tmp/wowp-066.ppm
/tmp/wowp-067.ppm
/tmp/wowp-068.ppm
/tmp/wowp-069.ppm
Command succeeded. Exit code: 0. Duration: 1386 ms.
```

search · Brewster Dinneen Faber gravity matrix graffiti eigenvalues

```
Web search results for "Brewster Dinneen Faber gravity matrix graffiti eigenvalues":
[1] A computational attack on the conjectures of Graffiti
URL: https://www.cs.auckland.ac.nz/~mjd/graffiti/graffiti1.pdf
Highlights: Tony L. Brewster, Michael J. Dinneen*, Vance Faber
...
Conjecture 179. The inverse transmission of gravity matrix  $\sim$  the average transmission of gravity matrix (connected graphs).
...
... 91 lines
Web search results for "Brewster Dinneen Faber gravity matrix graffiti eigenvalues":
[1] A computational attack on the conjectures of G ...
```

fetch · https://www.math.ucdavis.edu/~saito/data/distmat/aouchiche-hansen_dist-spec-graphs-survey.pdf

```
Fetches https://www.math.ucdavis.edu/~saito/data/distmat/aouchiche-hansen_dist-spec-graphs-survey.pdf - Distance spectra of
graphs: A survey
Content:
Linear Algebra and its Applications 458 (2014) 301–386
Contents lists available at ScienceDirect
#### Linear Algebra and its Applications
www.elsevier.com/locate/laa
... 142 lines
Fetches https://www.math.ucdavis.edu/~saito/data/distmat/aouchiche-hansen_dist-spec-graphs-survey.pdf - Distance spectra o ...
```

13 · 18:17

Let me OCR the WotW pages to find the conjectures. I'll convert ppm to png and run tesseract on a range of pages. Also let me try to get Aouchiche-Hansen survey via sci-hub properly.

\$ bash

```
# Convert more pages and OCR looking for conjecture numbers 284 etc
# First OCR the sample pages to see numbering
```

```
for f in /tmp/wowp-06{0..9}.ppm; do
  out=$(tesseract "$f" stdout 2>/dev/null | tr '\n' ' ' | head -c 400)
  echo "=== $(basename $f) ==="
  echo "$out"
  echo
done
```

```
=== wowp-060.ppm ===
```

not in the sense that they would solve all such problems (obviously!), but in the sense that they make no reference to the semantics of the domain of the data. On the other hand, the KEKADA program, which designs sequences of experiments to find the solution to a scientific problem (e.g., the chemical path of urea synthesis in vivo, or the induction of electricity from magnetism), requires both

```
=== wowp-061.ppm ===
```

reason why no one becomes a world-class expert in any domain without ten years of intense study and practice in that domain – no universal experts! I am not sure that these remarks are relevant to our earlier exchange, but in any event, I hope they throw some light on my views about the respective roles of the general and the domain-specific in discovery.” Cordially Herb Simon Concerning th

```
=== wowp-0 ...
```

14 · 18:17

Good - OCR works. Conjectures around 284 would be later. Let me figure out page mapping. Page 65 has conjecture 128. So roughly each page has ~10-15 conjectures. 284-128=156, so about 10-15 more pages...

OCR is working — jumping to pages covering 284–295.

```
$ bash
```

```
pdftoppm -f 75 -l 95 -r 160 /home/scout/conjecture-hunt/ce10-graffiti-girth/wow-july2004.pdf /tmp/wowq
for f in /tmp/wowq-*.ppm; do
  out=$(tesseract "$f" stdout 2>/dev/null)
  # show if contains 28x or 29x
  if echo "$out" | rg -q '\b(28[0-9]|29[0-9]|30[0-9])\b'; then
    echo "=== $(basename $f) ==="
    echo "$out"
    echo
  fi
done
```

```
=== wowq-079.ppm ===
```

279. If girth is > 5 then the matching number $<$ the sum of inverses of the rainbow.

280. If girth is > 5 then the mean of coordinates of mismatching $<$ matching.

281. max (matching, mismatching) number of negative eigenvalues of Distance. Michael J. Dinneen, Los Alamos National Laboratory and University of Victoria, Victoria, B.C (comp. 107.) August 91.

282. If girth is > 5 then the n - the independence number $<$ rank of the distance matrix.

283. If girth is > 5 then the independence $<$ number of nonpositive eigenvalues

... 78 lines

kkk

81

Command succeeded. Exit code: 0. Duration: 10799 ms.

15 · 18:17

Excellent! I have the exact statements:

I think 295 is the most vulnerable here. The bound on the number of positive distance eigenvalues feels like it could be the loosest one—especially for regular graphs where the distance matrix spectrum tends to spread out more than the adjacency one. For girth ≥ 5 graphs, the distance eigenvalues can get quite positive in the 0-to- n range, and $n/\text{mean}(\text{gravity})$ grows with $n^3/(d^2 \sum 1/\text{dist})$, which for cage graphs with small girth constraints might not cap the positive count tightly enough.

Statements locked. Implementing the spectral attack and testing cages plus random girth ≥ 5 graphs.

```
write → /home/scout/conjecture-hunt/ce10-graffiti-girth/attack.py
```

```
#!/usr/bin/env python3
"""
Finite counterexample hunt: Graffiti 284, 292, 295 (girth  $\geq 5$ ).
Sources: Written on the Wall (OCR of wow-july2004.pdf pp.79-80);
Roucairol-Cazenave arXiv:2409.18626 + refutationGBR score functions;
Brewster-Dinneen-Faber gravity matrix definition.
```

```

Definitions (documented):
    dual_degree(v) = mean degree of neighbors of v
    min_dual = min_v dual_degree(v)
    Gravity Gr[u,v] = 0 if u=v else (1/(n-1)) * deg(u)*deg(v) / dist(u,v)
        (Brewster/WotW; NOT Aouchiche-Hansen product-with-distance variant)
    mean_gravity = mean of ALL n^2 matrix entries (incl. diagonal 0) [nalgebra .mean()]
    range is NOT used in 284/292/295.
    "positive eigenvalue" means > 1e-8 (numerical).

Statements (WotW numbering; girth ≥ 5, connected, simple):
    284: min_dual ≤ - lambda_min(Distance)
    292: min positive adjacency eigenvalue ≤ n / mean_gravity
    295: (# positive distance eigenvalues) ≤ n / mean_gravity
        (Aouchiche-Hansen labels this 294)

Score (positive ⇒ counterexample):
    284: min_dual + lambda_min(D)
    292: min_pos_adj - n/mean_G
    295: num_pos_dist - n/mean_G
"""

from __future__ import annotations

import itertools
import math
import random
import sys
from dataclasses import dataclass
from typing import Iterable

import networkx as nx
import numpy as np

EPS = 1e-8

def girth(G: nx.Graph) → int | float:
    try:
        return nx.girth(G)
    except Exception:
        # networkx may raise on forests
        if nx.is_forest(G):
            return math.inf
        raise

def assert_girth_ge5(G: nx.Graph) → None:
    g = girth(G)
    if g < 5:
        raise ValueError(f"girth {g} < 5")

def degrees(G: nx.Graph, nodes: list) → np.ndarray:
    return np.array([G.degree(v) for v in nodes], dtype=float)

def dual_degrees(G: nx.Graph, nodes: list) → np.ndarray:
    out = []
    for v in nodes:
        nbrs = list(G.neighbors(v))
        if not nbrs:
            out.append(float("nan"))
        else:
            out.append(sum(G.degree(u) for u in nbrs) / len(nbrs))
    return np.array(out, dtype=float)

def adj_matrix(G: nx.Graph, nodes: list) → np.ndarray:
    return nx.to_numpy_array(G, nodelist=nodes, dtype=float)

def dist_matrix(G: nx.Graph, nodes: list) → np.ndarray:
    # floyd from networkx shortest_path_length
    n = len(nodes)

```

```

idx = {v: i for i, v in enumerate(nodes)}
D = np.zeros((n, n), dtype=float)
lengths = dict(nx.all_pairs_shortest_path_length(G))
for u in nodes:
    for v, d in lengths[u].items():
        D[idx[u], idx[v]] = float(d)
if np.any(D == 0) and n > 1:
    # disconnected if off-diagonal zero remains
    off = D + np.eye(n)
    if np.any(off == 0):
        raise ValueError("graph not connected")
return D

def gravity_matrix(G: nx.Graph, nodes: list) → np.ndarray:
    n = len(nodes)
    deg = degrees(G, nodes)
    D = dist_matrix(G, nodes)
    Gr = np.zeros((n, n), dtype=float)
    inv = 1.0 / (n - 1)
    for i in range(n):
        for j in range(n):
            if i == j:
                continue
            Gr[i, j] = inv * deg[i] * deg[j] / D[i, j]
    return Gr

def eigs_sym(M: np.ndarray) → np.ndarray:
    return np.linalg.eigvalsh(M)

```

```

@dataclass
class Report:
    name: str
    n: int
    m: int
    girth: float
    min_dual: float
    lam_min_D: float
    score284: float
    min_pos_A: float
    mean_G: float
    n_over_meanG: float
    score292: float
    num_pos_D: int
    score295: float
    edges: list[tuple]

```

... 456 lines

16 · 18:18

Let me run the attack script. Also note WotW OCR said "girth is > 5" which might be ≥ 5 . Petersen has girth 5 so if literally > 5, Petersen wouldn't apply - but Roucairol tests $\text{girth} \geq 5$ and Petersen ...

\$ bash

```

cd /home/scout/conjecture-hunt/ce10-graffiti-girth && /tmp/venv/bin/pip install networkx numpy -q && /tmp/venv/bin/python
attack.py 2>&1

```

```

skip McGee: module 'networkx' has no attribute 'mcgee_graph'
skip Tutte-Coxeter: module 'networkx' has no attribute 'tutte_coxeter_graph'
skip FlowerSnark5: module 'networkx' has no attribute 'flower_snark'
skip FlowerSnark7: module 'networkx' has no attribute 'flower_snark'
skip OddGraph4: module 'networkx' has no attribute 'odd_graph'
=== Named / cage / incidence graphs ===
C5              n= 5 m= 5 g=5 284=-0.618034 (du=2.0000 lminD=-2.6180) 292=-7.715299 (mpA=0.6180 n/mG=8.3333)
295=-7.333333 (#pD=1 n/mG=8.3333)
C6              n= 6 m= 6 g=6 284=-2.000000 (du=2.0000 lminD=-4.0000) 292=-12.500000 (mpA=1.0000 n/mG=13.5000)
295=-12.500000 (#pD=1 n/mG=13.5000)

```

```
C7                                n= 7 m= 7 g=7 284=-3.048917 (du=2.0000 lminD=-5.0489) 292=-18.798475 (mpA=1.2470 n/mG=20.0455)
295=-19.045455 (#pD=1 n/m --
```

17 · 18:19

Fantastic! Hoffman-Singleton is a counterexample to Graffiti 284!

Hoffman-Singleton is indeed a striking counterexample to Conjecture 284, and your spectral breakdown nails the exact values: the distance matrix D has eigenvalues 91 (multiplicity 1) and -4 (multiplicity 28), with the remaining 21 eigenvalues at 1, confirming $\lambda_{\min}(D) = -4$ exactly. For a 7-regular graph, the dual degree is uniformly 7, so the margin of 3 is precise and verifiable without approximation.

Hoffman-Singleton refutes 284 with exact margin 3. Writing a clean certificate and checking girth>5 variants.

\$ bash

```
/tmp/venv/bin/python << 'PY'
import networkx as nx, numpy as np
G = nx.hoffman_singleton_graph()
print('n', G.order(), 'm', G.size(), 'regular', set(dict(G.degree()).values()), 'girth', nx.girth(G), 'diam', nx.diameter(G))
# strongly regular params
A = nx.to_numpy_array(G, dtype=float)
eigA = np.linalg.eigvalsh(A)
print('A eigs unique rounded:', sorted(set(np.round(eigA,10))))
# D = 2J-2I-A since diam 2
n=50
J=np.ones((n,n)); I=np.eye(n)
D=2*J-2*I-A
eigD=np.linalg.eigvalsh(D)
print('D eigs unique rounded:', sorted(set(np.round(eigD,10))))
print('lam_min_D', eigD.min())
print('min_dual', 7.0)
print('score284', 7+eigD.min())
# analytic: A: 7^1, 2^28, -3^21
# D on 1-perp: -2-mu -> mu=2 => -4; mu=-3 => 1; on allones: 91
print('analytic lam_min=-4, score=7-4=3')
# Petersen equality
P=nx.petersen_graph()
A=nx.to_numpy_array(P,dtype=float)
# diam2: D=2J-2I-A
n=10; D=2*np.ones((n,n))-2*np.eye(n)-A
print('Petersen lam_min_D', np.linalg.eigvalsh(D).min(), 'score', 3+np.linalg.eigvalsh(D).min())
PY
# also hunt girth>=6 high degree for robustness
/tmp/venv/bin/python << 'PY'
import networkx as nx, numpy as np, random
from attack import evaluate, random_regular_girth, incidence_projective_plane, girth

# known (3,g) cages missing from nx - build Tutte-Coxeter manually?
# Try Robertson graph etc.
... 30 lines
```

```
n 50 m 175 regular {7} girth 5 diam 2
A eigs unique rounded: [np.float64(-3.0), np.float64(2.0), np.float64(7.0)]
D eigs unique rounded: [np.float64(-4.0), np.float64(1.0), np.float64(91.0)]
lam_min_D -4.0000000000000017
min_dual 7.0
score284 2.9999999999999983
analytic lam_min=-4, score=7-4=3
Petersen lam_min_D -3.0000000000000018 score -1.7763568394002505e-15
Traceback (most recent call last):
  File "<stdin>", line 2, in <module>
ModuleNotFoundError: No module named 'attack'
Command failed. Exit code: 1. Duration: 246 ms.
```

\$ bash

```
# Build McGee and Tutte-Coxeter from known edge lists if needed
/tmp/venv/bin/python << 'PY'
import networkx as nx
print(nx.__version__)
print([x for x in dir(nx) if 'cage' in x.lower() or 'tutte' in x.lower() or 'mcgee' in x.lower() or 'heawood' in x.lower() or
'hoffman' in x.lower() or 'sargent' in x.lower() or 'balaban' in x.lower()])
# appollonian?
```

```
print([x for x in dir(nx) if 'graph' in x.lower() and any(k in x.lower() for k in
['moebius', 'pappus', 'desargues', 'franklin', 'dyck', 'foster', 'nauru', 'sylveste'])])
PY
```

```
3.6.1
['heawood_graph', 'hoffman_singleton_graph', 'tutte_graph', 'tutte_polynomial']
['desargues_graph', 'moebius_kantor_graph', 'pappus_graph']
Command succeeded. Exit code: 0. Duration: 90 ms.
```

18 · 18:20

There's also desargues_graph, moebius_kantor, pappus. Let me evaluate those and write the final certificate. Also fix the girth ≥ 6 hunt by running from the right directory.

\$ bash

```
cd /home/scout/conjecture-hunt/ce10-graffiti-girth && /tmp/venv/bin/python << 'PY'
import networkx as nx
from attack import evaluate, fmt, girth, random_regular_girth

for name, G in [
    ('Desargues', nx.desargues_graph()),
    ('Moebius-Kantor', nx.moebius_kantor_graph()),
    ('Pappus', nx.pappus_graph()),
    ('Tutte', nx.tutte_graph()),
]:
    g = girth(G)
    print(name, 'n', G.order(), 'g', g, 'conn', nx.is_connected(G))
    if g  $\geq$  5 and nx.is_connected(G):
        print(fmt(evaluate(G, name)))

best=None
for n in list(range(14,81,2)):
    for d in [3,4,5]:
        if n*d % 2: continue
        for s in range(12):
            G=random_regular_girth(n,d,seed=17000+n*50+d*10+s,tries=60)
            if G is None: continue
            gg=girth(G)
            if gg < 6: continue
            r=evaluate(G,f'g6RR({n},{d})g={gg}#{s}')
            if best is None or r.score284>best.score284:
                best=r
                print('best g $\geq$ 6', fmt(r))
print('FINAL best g $\geq$ 6 284:', None if best is None else (best.name, best.score284))
PY
```

```
Desargues n 20 g 6 conn True
Desargues          n= 20 m= 30 g=6 284=-9.000000 (du=3.0000 lminD=-12.0000) 292=-93.351335 (mpA=1.0000
n/mG=94.3513) 295=-93.351335 (#pD=1 n/mG=94.3513)
Moebius-Kantor n 16 g 6 conn True
Moebius-Kantor     n= 16 m= 24 g=6 284=-4.464102 (du=3.0000 lminD=-7.4641) 292=-52.894737 (mpA=1.0000 n/mG=53.8947)
295=-49.894737 (#pD=4 n/mG=53.8947)
Pappus n 18 g 6 conn True
Pappus             n= 18 m= 27 g=6 284=-4.464102 (du=3.0000 lminD=-7.4641) 292=-70.267949 (mpA=1.7321 n/mG=72.0000)
295=-67.000000 (#pD=5 n/mG=72.0000)
Tutte n 46 g 4 conn True
best g $\geq$ 6 g6RR(48,3)g=6#7          n= 48 m= 72 g=6 284=-33.985486 (du=3.0000 lminD=-36.9855) 292=-767.972813 (mpA=0.0953
n/mG=768.0681) 295=-749.068091 (#pD=19 n/mG=768.0681)
best g $\geq$ 6 g6RR(50,3)g=6#9          n= 50 m= 75 g=6 284=-33.452832 ...
```

19 · 18:21

Girth ≥ 6 seems much harder for 284 - margins go more negative. The Hoffman-Singleton with girth 5 is the clear counterexample for the standard ≥ 5 interpretation.

Writing the certificate and final report.

write → /home/scout/conjecture-hunt/ce10-graffiti-girth/cert_284_hoffman_singleton.py

```
#!/usr/bin/env python3
"""Exact certificate: Hoffman-Singleton refutes Graffiti 284."""
from __future__ import annotations
```

```

import networkx as nx
import numpy as np

# Graffiti 284 (Written on the Wall; girth  $\geq 5$ ):
#  $\min \text{dual degree} \leq -\lambda_{\min}(\text{Distance})$ 
# Dual degree of  $v$  = mean degree of neighbors of  $v$ .
# For  $d$ -regular graphs,  $\min \text{dual degree} = d$ .

G = nx.hoffman_singleton_graph()
assert G.order() == 50
assert G.size() == 175
assert nx.is_connected(G)
assert nx.girth(G) == 5
assert nx.diameter(G) == 2
assert set(dict(G.degree()).values()) == {7}

# HS is strongly regular  $\text{srg}(50, 7, 0, 1)$ . Adjacency spectrum (exact):
# 7 (mult 1), 2 (mult 28), -3 (mult 21)
A = nx.to_numpy_array(G, dtype=float)
eigs_A = np.linalg.eigvalsh(A)
assert abs(eigs_A.max() - 7) < 1e-9
# multiplicities
def mult(val, tol=1e-8):
    return int(np.sum(np.abs(eigs_A - val) < tol))
assert mult(7) == 1 and mult(2) == 28 and mult(-3) == 21

# Diameter 2  $\Rightarrow$  Distance  $D = 2(J - I) - A = 2J - 2I - A$ 
n = 50
D = 2 * np.ones((n, n)) - 2 * np.eye(n) - A

# Spectrum of D (exact, from A-spectrum):
# On all-ones:  $D \cdot 1 = (100 - 2 - 7) \cdot 1 = 91 \cdot 1$ 
# On 1-perp with A-eigenvalue  $\mu$ :  $D \cdot v = (-2 - \mu) \cdot v$ 
#  $\mu=2 \Rightarrow -4$  (mult 28)
#  $\mu=-3 \Rightarrow 1$  (mult 21)
lam_min_D = -4.0
min_dual = 7.0 # 7-regular
lhs = min_dual
rhs = -lam_min_D # = 4
score = lhs - rhs # = 3 > 0  $\Rightarrow$  counterexample

# numerical check
eigs_D = np.linalg.eigvalsh(D)
assert abs(eigs_D.min() - (-4)) < 1e-9
assert abs(eigs_D.max() - 91) < 1e-9

print("Graffiti 284 COUNTEREXAMPLE: Hoffman-Singleton graph HS(50)")
print(f" n={n}, m=175, 7-regular, girth=5, diameter=2")
print(f" min dual degree = {min_dual}")
print(f"  $\lambda_{\min}(\text{Distance}) = \{\text{lam\_min\_D}\}$  (exact)")
print(f"  $-\lambda_{\min}(D) = \{\text{rhs}\}$ ")
print(f" conjecture claims:  $\{\text{lhs}\} \leq \{\text{rhs}\}$ ")
print(f" FALSE with margin score =  $\text{min\_dual} + \text{lam\_min}(D) = \{\text{score}\}$ ")
print()
print("Sharp equality case: Petersen (3-regular, girth 5, diam 2)")
print(f" min_dual=3,  $\text{lam\_min}(D)=-3$ , score=0 (equality)")
print()
print("CERTIFICATE OK")

```

write $\rightarrow$ /home/scout/conjecture-hunt/ce10-graffiti-girth/REPORT.md

```

# Graffiti 284 / 292 / 295 – finite counterexample hunt

Work dir: `/home/scout/conjecture-hunt/ce10-graffiti-girth/`

Sources: Written on the Wall (wow-july2004.pdf, OCR pp. 79–80);
Roucairol-Cazenave arXiv:2409.18626 (open, girth  $\geq 5$ );
Roucairol `refutationGBR` score functions;
Brewster-Dinneen-Faber gravity matrix.

## Statements (WotW; connected simple graphs with girth  $\geq 5$ )

| ID | Statement |

```

```

|----|-----|
| **284** | min dual degree  $\leq -\lambda_{\min}(\text{Distance})$  |
| **292** | min positive adjacency eigenvalue  $\leq n / \text{mean}(\text{Gravity})$  |
| **295** | ( $\#$  positive distance eigenvalues)  $\leq n / \text{mean}(\text{Gravity})$  |

Aouchiche-Hansen labels **295** as **294**.

### Definitions used

- **dual degree** of  $v$ : mean of degrees of neighbors of  $v$ .  $\text{min dual}$  = min over vertices.
- **Gravity matrix** (Brewster / WotW, NOT the A&H product-with-distance variant):
   $\text{Gr}[u,v] = 0$  if  $u=v$ , else  $(1/(n-1)) * \text{deg}(u) * \text{deg}(v) / \text{dist}(u,v)$ .
- **mean(Gravity)**: arithmetic mean of all  $n^2$  entries (including diagonal zeros), matching nalgebra .mean() in Roucairol's code.
- **positive eigenvalue**:  $> 1e-8$  numerically.
- **range**: not used in 284/292/295. (Elsewhere in Graffiti/A&H, "range" is contested: max-min vs  $\#$  distinct; irrelevant here.)
- **girth  $\geq 5$ **: Roucairol table + standard Graffiti reading of "girth  $> 5 / \geq 5$ " on cages. Petersen is the equality case of 284, so girth 5 is included.

## Most vulnerable: **284**

Simplest invariants (one dual-degree scalar + one distance eigenvalue). Sharp on Petersen:

- Petersen: 3-regular, girth 5,  $\text{diam } 2 \rightarrow D = 2J - 2I - A$ 
- A-spectrum:  $3, 1^5, (-2)^4 \rightarrow D$ -spectrum:  $15, 1^4, (-3)^5$ ? wait  $\text{diam}=2$  formula on 1-perp:  $-2-\mu$ 
  -  $\mu=1 \rightarrow -3$  (mult 5);  $\mu=-2 \rightarrow 0$  (mult 4); on ones: 15
  - Actually numerical:  $\lambda_{\min}(D)=-3$ ,  $\text{min\_dual}=3$ ,  $\text{score} = 0$  (equality)**.

## COUNTEREXAMPLE to 284: Hoffman-Singleton

**Hoffman-Singleton graph** HS: unique (7,5)-cage,  $\text{srg}(50,7,0,1)$ .

| quantity | value |
|-----|-----|
| n, m | 50, 175 |
| degree | 7-regular |
| girth | 5 |
| diameter | 2 |
| A-spectrum |  $7^1, 2^{28}, (-3)^{21}$  |
|  $D = 2J - 2I - A$  | spectrum  $91^1, 1^{21}, (-4)^{28}$  |
| min dual degree | 7 |
|  $-\lambda_{\min}(D)$  | 4 |
|  $**7 \leq 4?$  | **FALSE** |
| margin  $\text{'min\_dual'} + \lambda_{\min}(D)$  |  $**+3$  |

Exact proof (no numerics required beyond the classical HS adjacency spectrum):

1. HS is 7-regular  $\Rightarrow$  min dual degree = 7.
2.  $\text{diam}(\text{HS})=2 \Rightarrow D = 2J - 2I - A$ .
3. On  $1^A$ , eigenvalue of  $D$  is  $-2 - \mu$  for each A-eigenvalue  $\mu$ .
4.  $\mu = 2$  (mult 28)  $\Rightarrow D$ -eigenvalue  $-4$ .
5. So  $\lambda_{\min}(D) = -4$ , and  $7 \neq 4$ .

Run:



```

'''bash
python3 cert_284_hoffman_singleton.py
python3 attack.py # broad search; finds HS immediately among named graphs
'''

292 and 295

No counterexample found. Best (least negative) margins on the tested family:

conj	best score	graph	note
292	≈ -7.72	C5	$\text{min_pos_A}=0.618, n/\text{mean_G}=8.333$
295	≈ -7.33	C5	$\# \text{pos_D}=1, n/\text{mean_G}=8.333$

On cages / projective-plane incidence graphs / random regular $\text{girth} \geq 5$ up to $n=50$, both stay largely negative (RHS $n/\text{mean_G}$ grows fast). Not refuted here.

Files

```


```

- `cert\_284\_hoffman\_singleton.py` - exact CE certificate
- `attack.py` - spectral scanner (cages, PG(2,q) incidence, random regular/6NM, local search)
- `wow-july2004.pdf` - Written on the Wall snapshot used for OCR of statements
- `results.txt` - best margins dump

\$ bash

```
cd /home/scout/conjecture-hunt/ce10-graffiti-girth && /tmp/venv/bin/python cert_284_hoffman_singleton.py &&
/tmp/venv/bin/python -c "
import networkx as nx, numpy as np
# double-check Petersen equality exactly
P=nx.petersen_graph()
A=nx.to_numpy_array(P,dtype=float)
n=10
D=2*np.ones((n,n))-2*np.eye(n)-A
print('Petersen D eigs', np.round(np.linalg.eigvalsh(D),10))
print('score', 3+np.linalg.eigvalsh(D).min())
"
ls -la /home/scout/conjecture-hunt/ce10-graffiti-girth/
```

```
Graffiti 284 COUNTEREXAMPLE: Hoffman-Singleton graph HS(50)
n=50, m=175, 7-regular, girth=5, diameter=2
min dual degree = 7.0
lambda_min(Distance) = -4.0 (exact)
-lambda_min(D) = 4.0
conjecture claims: 7.0 ≤ 4.0
FALSE with margin score = min_dual + lam_min(D) = 3.0
Sharp equality case: Petersen (3-regular, girth 5, diam 2)
min_dual=3, lam_min(D)=-3, score=0 (equality)
CERTIFICATE OK
... 10 lines
-rw-r--r-- 1 root root 907641 Jul 22 18:16 wow-july2004.pdf
-rw-r--r-- 1 root root 956674 Jul 22 18:16 wow.txt
Command succeeded. Exit code: 0. Duration: 257 ms.
```

20 · 18:21

I should fix a small error in the REPORT about Petersen D-spectrum - I had a confused intermediate calculation. The correct one is in the certificate: $\lambda_{\min}=-3$, zeros mult 4, 15 once... actually eigs:...

edit → /home/scout/conjecture-hunt/ce10-graffiti-girth/REPORT.md

```
- - Petersen: 3-regular, girth 5, diam 2 → `D = 2J-2I-A`
- A-spectrum: `3, 1^5, (-2)^4` → D-spectrum: `15, 1^4, (-3)^5`? wait diam-2 formula on 1-perp: `-2-μ`
- μ=1 → -3 (mult 5); μ=-2 → 0 (mult 4); on ones: 15
- Actually numerical: λ_min(D)=-3, min_dual=3, **score = 0 (equality)**
+ - Petersen: 3-regular, girth 5, diam 2 → `D = 2J-2I-A`
- A-spectrum: `3^1, 1^5, (-2)^4` → on `1^1`, D-eig = `-2-μ`: μ=1→-3 (x5), μ=-2→0 (x4); on ones: 15
- λ_min(D)=-3, min_dual=3, **score = 0 (equality)**
```

21 · 18:21

Deliver the final summary to the user.

****Graffiti 284 is false.**** Hoffman-Singleton is an exact finite counterexample (margin +3).

Statements (WotW; girth ≥ 5)

ID	Conjecture
284	min dual degree ≤ -λ _{min} (Distance)
292	min positive adj. eigenvalue ≤ n / mean(Gravity)
295	(# positive distance eigs) ≤ n / mean(Gravity) * (A&H #294)

****Gravity**** (Brewster/WotW): $\text{Gr}[u,v] = (1/(n-1)) \cdot \deg(u) \cdot \deg(v) / \text{dist}(u,v)$ (0 on diagonal). Mean = average of all n^2 entries. Dual degree(v) = mean degree of neighbors. Range is not used here.

Why 284

One scalar + one eigenvalue; ****sharp on Petersen**** (equality: $\min_dual=3, \lambda_min(D)=-3$).

Certificate — Hoffman–Singleton

HS = srg(50,7,0,1): 7-regular, girth 5, diam 2, A-spectrum $7^1, 2^{28}, (-3)^{21}$.

- diam 2 $\Rightarrow D = 2J - 2I - A$

- on $1^\perp$: D-eig = $-2 - \mu \Rightarrow \mu=2$ gives -4 ($\times 28$)

- min dual = 7, $-\lambda_min(D) = 4$

- $-4 \leq 4$ is false; score $\min_dual + \lambda_min(D) = +3$

292 / 295

No CE. Best margins still largely negative ($C5 \approx -7.7 / -7.3$). Cages, PG(2,q) incidence, random regular girth ≥ 5 up to $n=50$ all safe.

Files

`/home/scout/conjecture-hunt/ce10-graffiti-girth/`

- `cert_284_hoffman_singleton.py` — exact CE

- `attack.py` — full scan

- `REPORT.md` — statements + proof